intpic

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Outline

The GAP function DotFileLatticeSubgroups can be used to produce a graphical representation of the subgroup lattice of a group.
From the GAP manual:

```
g:=SymmetricGroup(4);;
l:=LatticeSubgroups(g);
DotFileLatticeSubgroups(l,"s4lat.dot");
```

The dot code is written into the file s4lat.dot. Then some tool can be used for the visualization. By using a slight modification (which currently is available in `www.fc.up.pt/cmup/mdelgado/files/latticedot_modified.g`) one can get the same result by doing

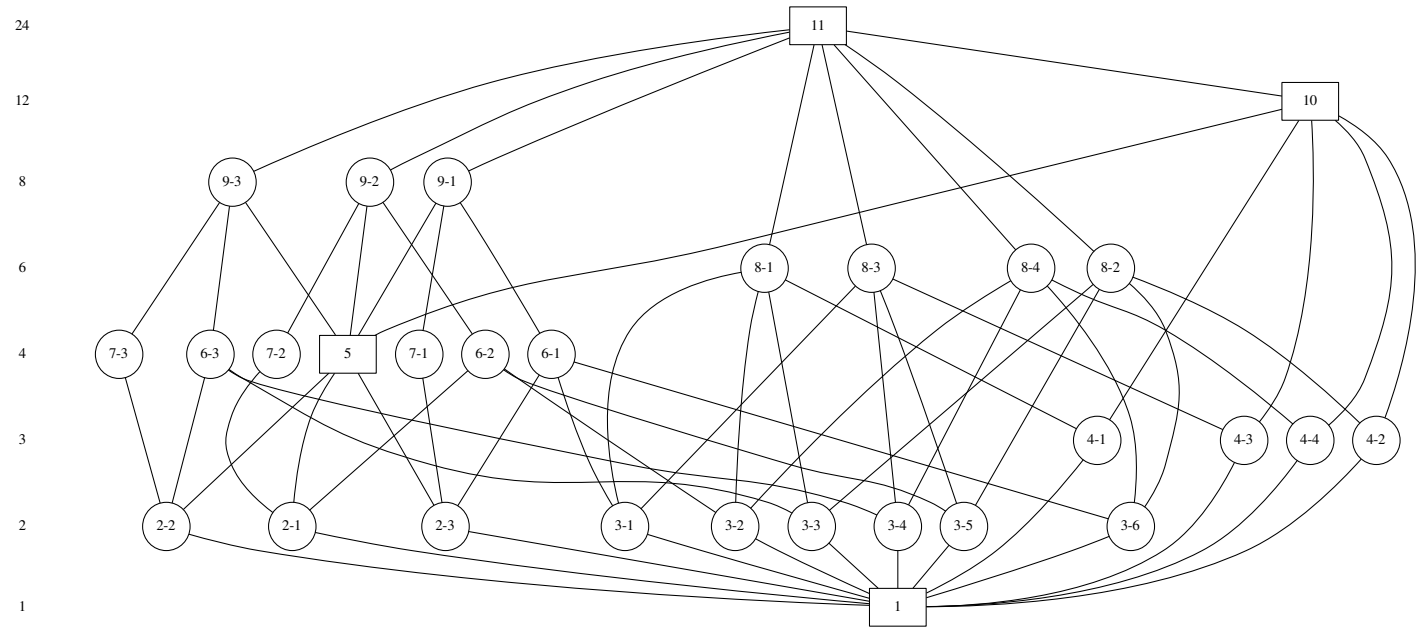
```
g:=SymmetricGroup(4);;
l:=LatticeSubgroups(g);
DotLatticeSubgroups(l);
PrintTo("s4lat.dot",last);
```

One can use the string produced to view the object just by typing

```
Splash(DotLatticeSubgroups(l));
```

where “Splash” is a funcion of the package “viz”

M. Delgado, A. Egri-Nagy, J. D. Mitchell and M. Pfeiffer. Viz - a GAP package for drawing GAP objects. Under development. https://bitbucket.org/zen154115/viz



For integers visualization...

M. Delgado.
IntPic – a GAP package for drawing integers, 2013, July. Version number 0.1.0.
`http://cmup.fc.up.pt/cmup/mdelgado/intpic/`

The following commands can be used in GAP to produce two lists of numbers...

```
twins := Filtered(Primes, p -> p + 2 in Primes);;
set := [0..999];;
arr := [Primes,Union(twins,twins+2)];;
```

Then, using intpic, one can obtain Tikz code (that can be processed with LaTeX) to produce a picture that highlights the numbers in these lists.

```
rowlength := 25;;
tkz := IP_TikzArrayOfIntegers(set,rowlength,rec(highlights:=arr))
```

The picture pops up by executing

```
IP_Splash(tkz);
```

975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999
950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974
925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949
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775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799
750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774
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600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624
575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599
550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574
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475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499
450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474
425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449
400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424
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300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324
275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299
250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274
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200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224
175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199
150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174
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75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74
25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

In case you do not like the red and green colors you may choose others.

```
cls := IP_ColorsRedTones;;
tkz := IP_TikzArrayOfIntegers(set,rowlength,
        rec(colors := cls,highlights:=arr));;

IP_Splash(tkz);
```

is just an abbreviation for

```
ltx := Concatenation("%tikz\n",IP_Preamble,tkz,IP_Closing);;
Splash(ltx);
```

975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999
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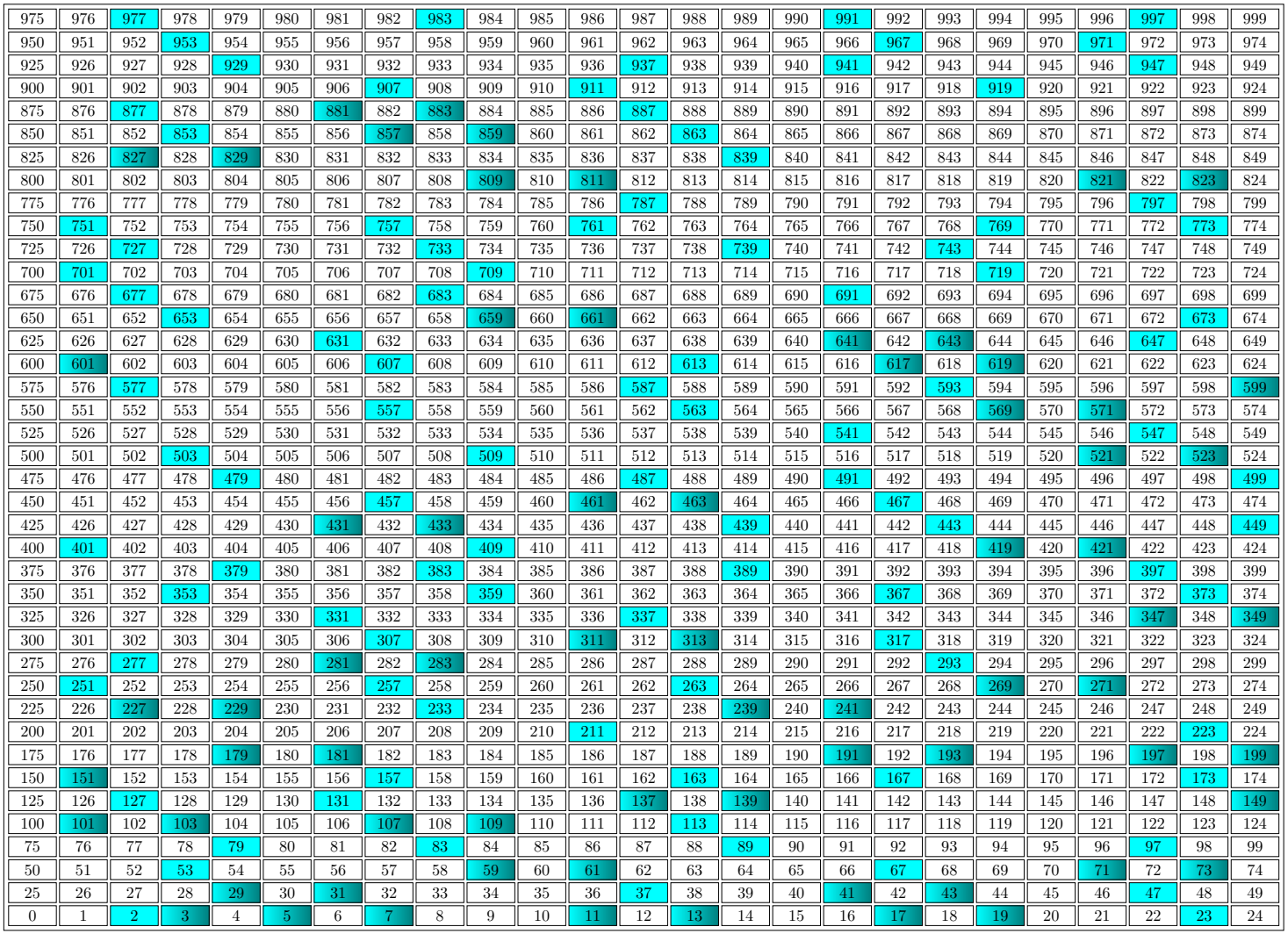
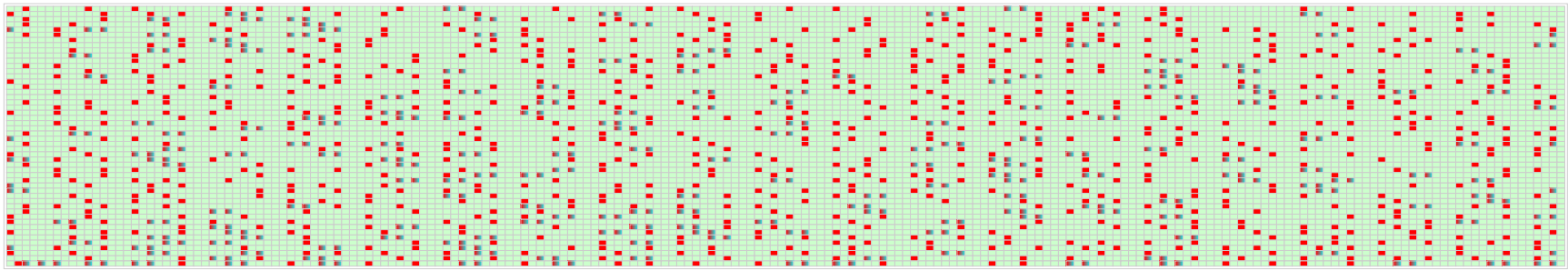
100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124
75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74
25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24

```
cls := IP_ColorsCompRedTones;;
tkz := IP_TikzArrayOfIntegers(set,rowlength,
       rec(colors := cls,highlights:=arr));;
```



With some more patience one can produce something like



Numerical semigroups

A **numerical semigroup** S is a submonoid of $\mathbb{N} = \mathbb{Z}_{\geq 0}$ (under addition) such that $\sharp(\mathbb{N} \setminus S) < \infty$.

Definitions and notation

Let S be a numerical semigroup.

- The elements of $\mathbb{N} \setminus S$ are said to be the **gaps** of S;
- $\mathbf{g} = \mathbf{g}(S) := \sharp(\mathbb{N} \setminus S)$ is the **genus** of S;
- the unique element $\mathbf{c} = \mathbf{c}(S) \in S$ such that $c(S) - 1 \notin S$ and $c(S) + \ell \in S$ for all $\ell \in \mathbb{N}$ is the **conductor** of S;
- the **Frobenius number** of S is the greatest integer $\mathbf{F}(\mathbf{S})$ not belonging to S (i.e., $F(S) = c(S) - 1$);
- the **multiplicity** of S is the least positive integer belonging to S.

A book...

 J. Rosales and P. García Sánchez. *Numerical Semigroups*. Springer, 2009.

For computations...

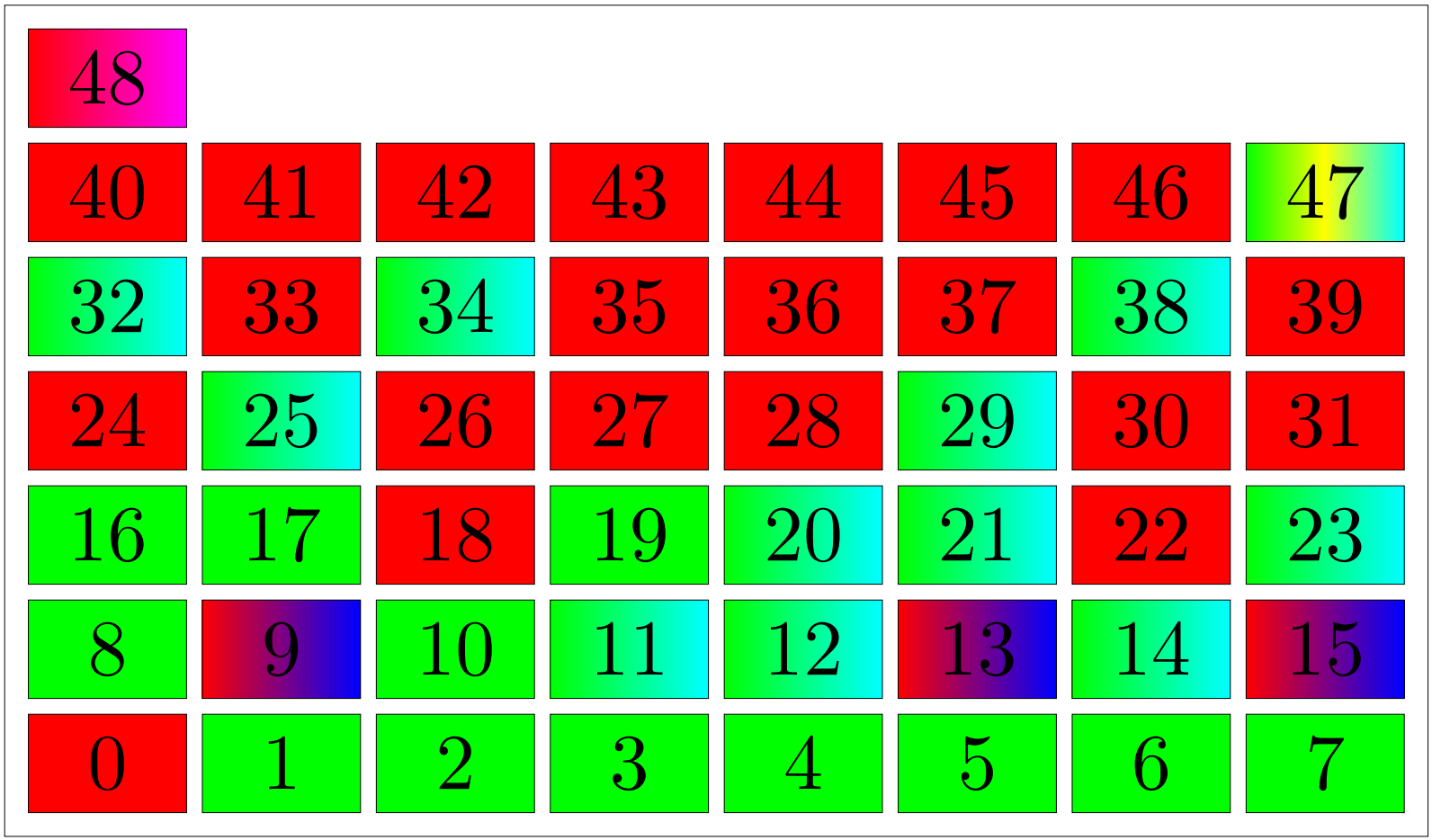
M. Delgado, J. Morais, and P. García-Sánchez. Numericalsgps – a GAP package for computing with numerical semigroups, 2013, June. Version number 0.980. http://cmup.fc.up.pt/cmup/mdelgado/numericalsgps/

An example of use

```
ns := NumericalSemigroup(9,13,15);
arr := [SmallElementsOfNumericalSemigroup(ns),
        GapsOfNumericalSemigroup(ns),
        MinimalGeneratingSystemOfNumericalSemigroup(ns),
        FundamentalGapsOfNumericalSemigroup(ns),
        [ConductorOfNumericalSemigroup(ns)],
        PseudoFrobeniusOfNumericalSemigroup(ns)];;
tkz := IP_TikzArrayOfIntegers(rec(highlights:=arr));;
```

A single row may not be the most convenient way to visualise

```
tkz := IP_TikzArrayOfIntegers(8,rec(highlights:=arr));;
```

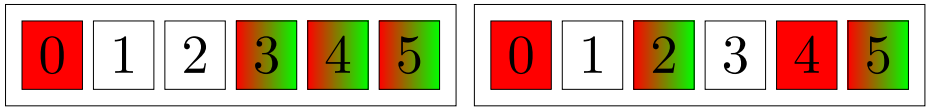


Counting numerical semigroups

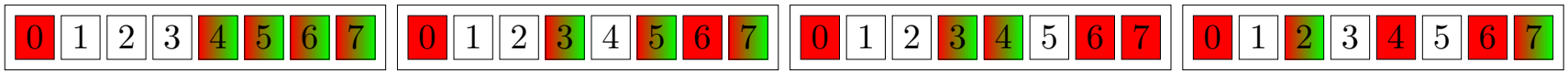
The genus of a numerical semigroup is the number of positive integers not belonging to it.

The numerical semigroups of genus $g+1$ can be obtained from the semigroups of genus g by removing a minimal generator. One can avoid repetitions by removing only generators that are greater than the Frobenius number.

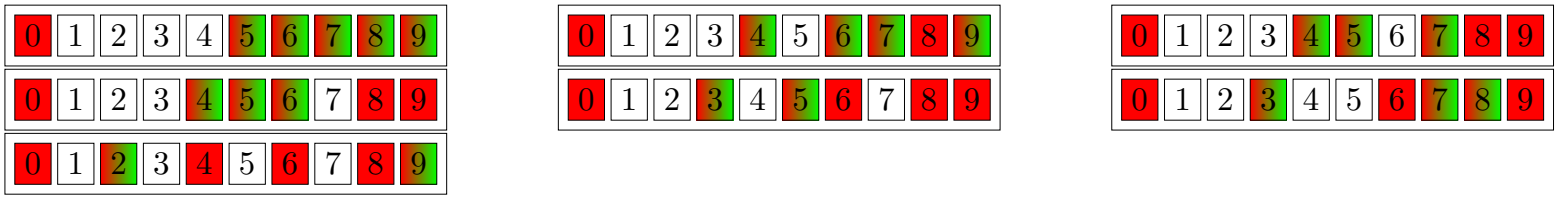
Genus 2



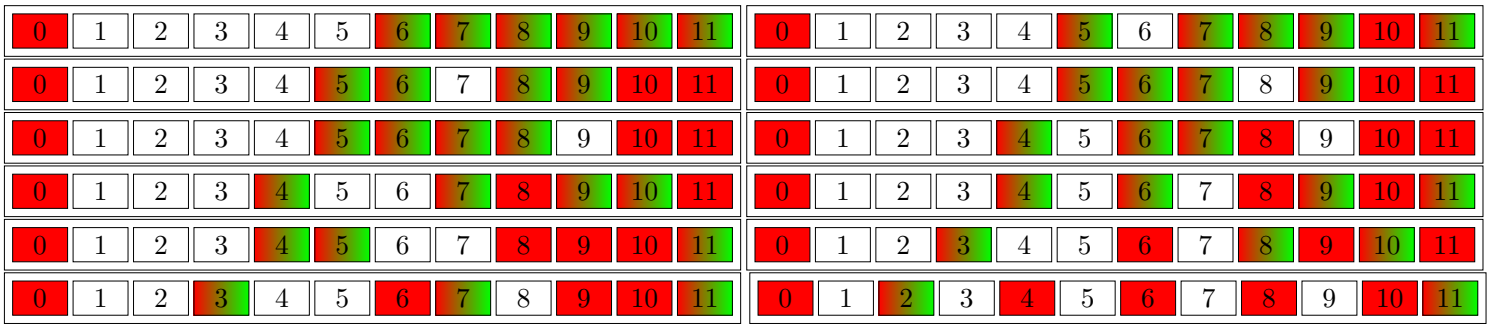
Genus 3



Genus 4



Genus 5



It is known that the sequence (n_g) of the number of numerical semigroups of genus g behaves like the Fibonacci sequence. It has therefore an exponential growth.


M. Bras-Amorós, Fibonacci-like behavior of the number of numerical semigroups of a given genus, Semigroup Forum, 76 (2008), 379–384.


Zhai, Alex. Fibonacci-like growth of numerical semigroups of a given genus, Semigroup Forum, 86 (2013) 634-662

What happens when one considers special families of numerical semigroups instead of all the numerical semigroups?

Storing sufficient data to made quickly available any numerical semigroup of a given genus is therefore unfeasible unless the genus is small. How small is certainly the result of a compromise between the available computational means and the users' needs...

For other problems...


M. Delgado, J.C. Rosales and P. A. García-Sánchez, *Numerical semigroups problem list*, International Center for Mathematics (CIM) Bulletin, 2013, number 33, 15–26;

Some more pictures

The Feng Rao numbers were studied infollowing pictures appeared in

M. Delgado, J. I. Farrán, P. A. García-Sánchez, and D. Llena.
On the weight hierarchy of codes coming from semigroups with two generators.
IEEE Trans. Inform. Theory, 60(1):282 – 295, 2014.

Some parts of the paper are rather technical. Detailed examination of a huge number of examples was crucial to form the ideas.
Some others (not shown here) allowed us to conclude that we were not following the right direction.

617	628	639	650	661	672	683	694	705	716	727	738	749	760	771	782	793	804	815	826	837	848	859	870	881	892	903	914	925
588	599	610	621	632	643	654	665	676	687	698	709	720	731	742	753	764	775	786	797	808	819	830	841	852	863	874	885	896
559	570	581	592	603	614	625	636	647	658	669	680	691	702	713	724	735	746	757	768	779	790	801	812	823	834	845	856	867
530	541	552	563	574	585	596	607	618	629	640	651	662	673	684	695	706	717	728	739	750	761	772	783	794	805	816	827	838
501	512	523	534	545	556	567	578	589	600	611	622	633	644	655	666	677	688	699	710	721	732	743	754	765	776	787	798	809
472	483	494	505	516	527	538	549	560	571	582	593	604	615	626	637	648	659	670	681	692	703	714	725	736	747	758	769	780
443	454	465	476	487	498	509	520	531	542	553	564	575	586	597	608	619	630	641	652	663	674	685	696	707	718	729	740	751
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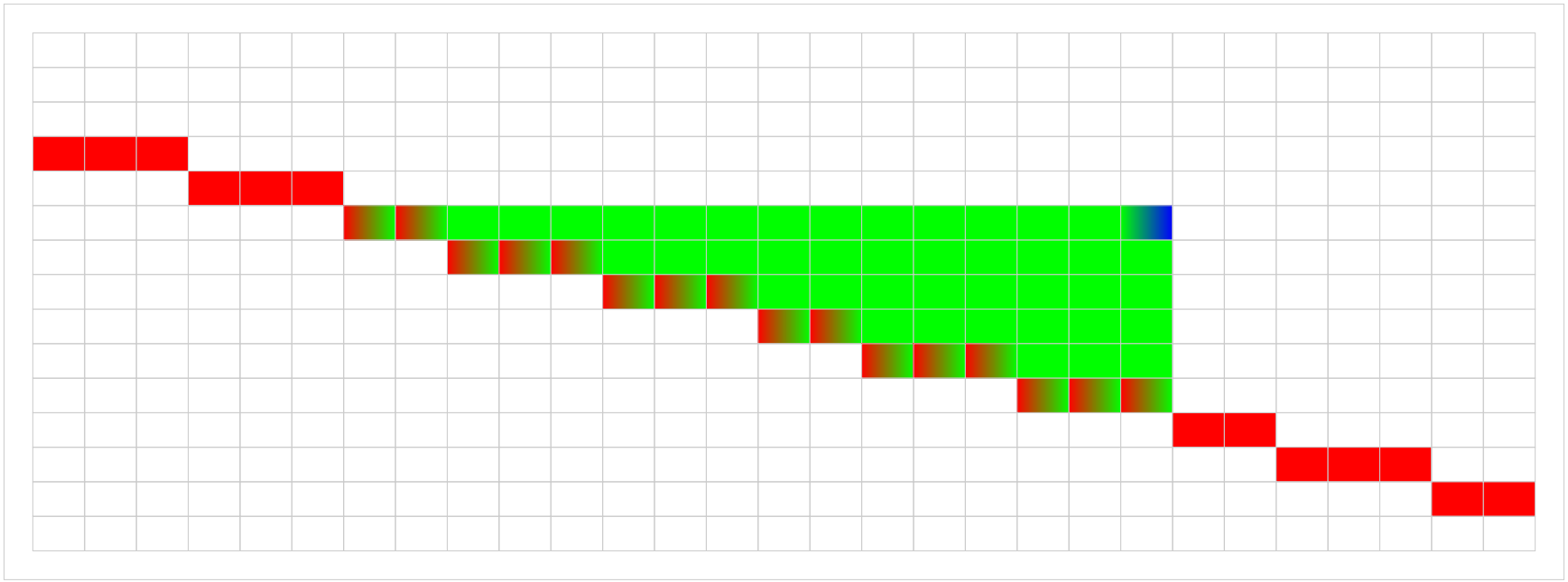
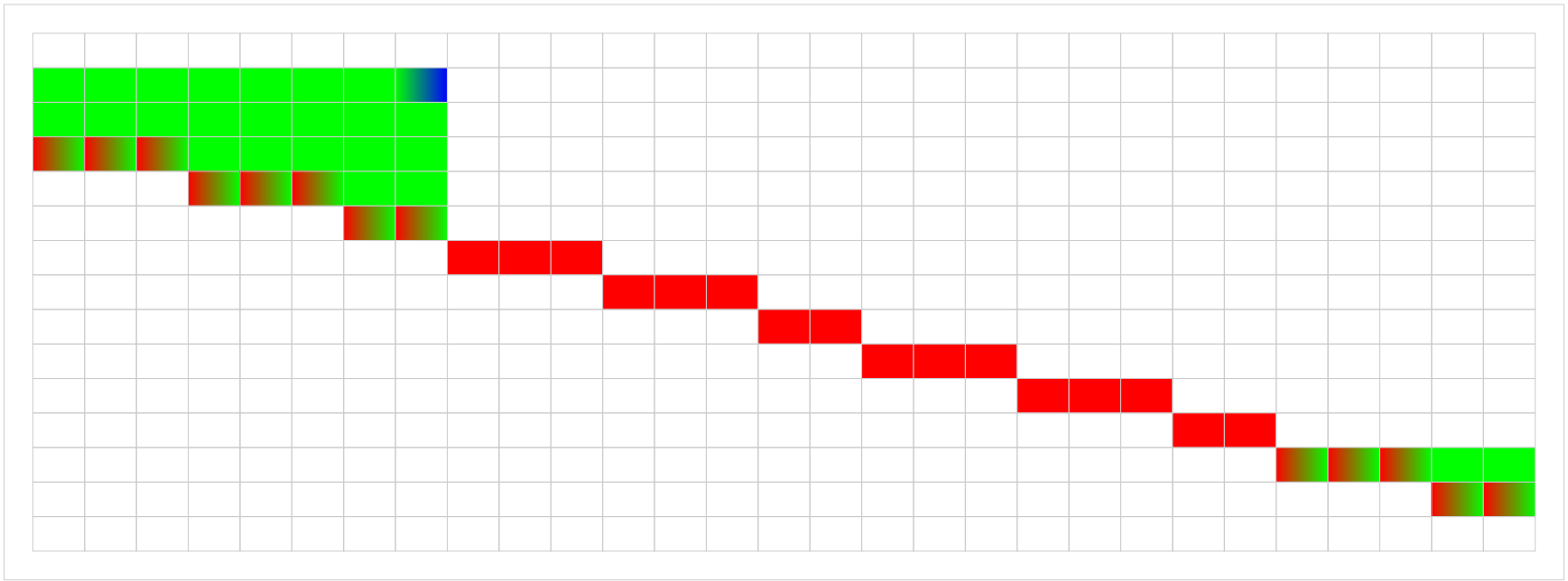
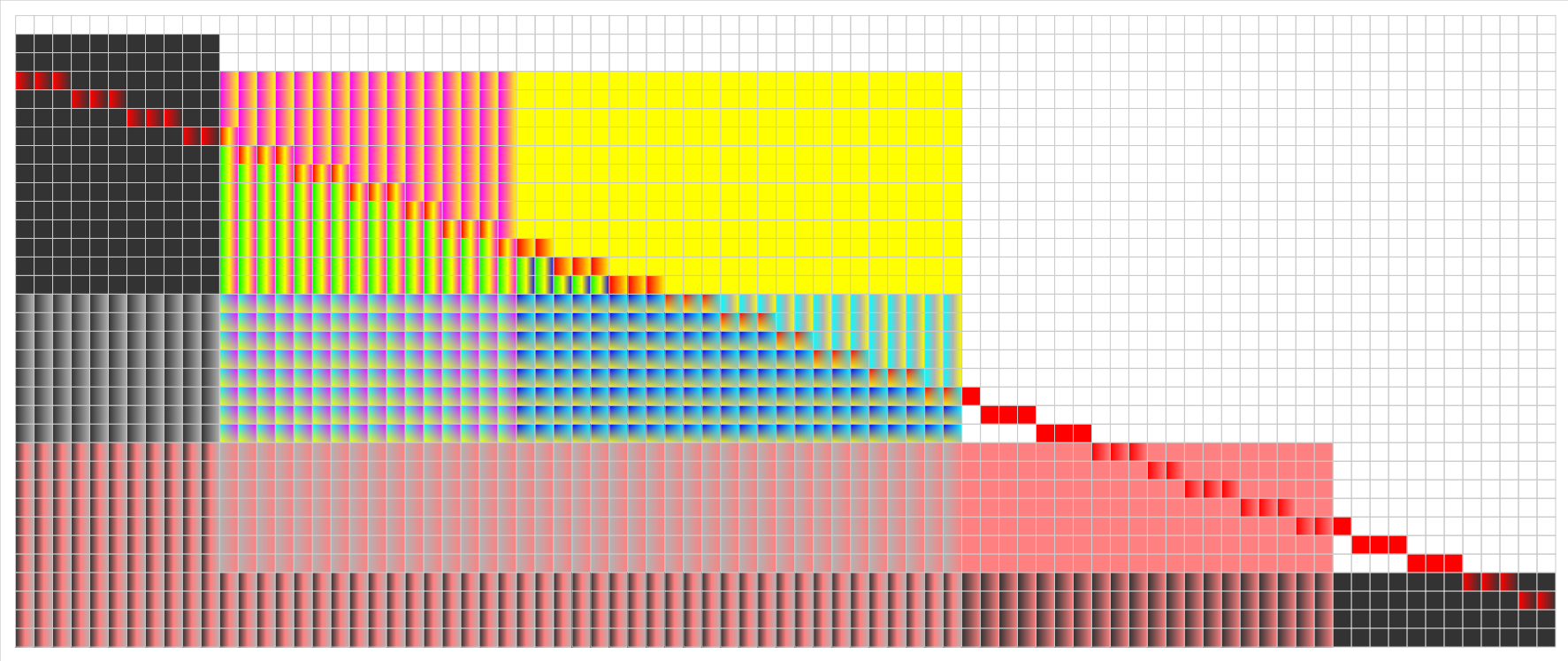
298	309	320	331	342	353	364	375	386	397	408	419	430	441	452	463	474	485	496	507	518	529	540	551	562	573	584	595	606
269	280	291	302	313	324	335	346	357	368	379	390	401	412	423	434	445	456	467	478	489	500	511	522	533	544	555	566	577
240	251	262	273	284	295	306	317	328	339	350	361	372	383	394	405	416	427	438	449	460	471	482	493	504	515	526	537	548
211	222	233	244	255	266	277	288	299	310	321	332	343	354	365	376	387	398	409	420	431	442	453	464	475	486	497	508	519





The following picture was used to help to follow the proof of the main technical lemma.

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