String theory, sheaf cohomology and the *homalg* package

Martin Bies

August 25, 2014

Martin Bies String theory, sheaf cohomology and the homalg package





1 Brief introduction to string theory





2 Sheaf cohomology in string theory and the homalg package

Section 1

Brief introduction to string theory

Getting started with string theory

Getting started with string theory

Why string theory?

• Current understanding of phsyics:

General relativity + Quantum field theory

Getting started with string theory

Why string theory?

• Current understanding of phsyics:

General relativity + Quantum field theory

• Is there a single description of everything?

Getting started with string theory

Why string theory?

• Current understanding of phsyics:

General relativity + Quantum field theory

- Is there a single description of everything?
- \Rightarrow String theory is a good candidate theory

Getting started with string theory

Why string theory?

• Current understanding of phsyics:

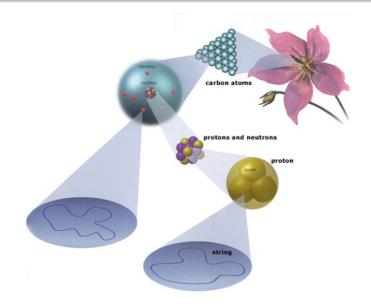
General relativity + Quantum field theory

- Is there a single description of everything?
- \Rightarrow String theory is a good candidate theory

Starting point of string theory:

Replace point particles by 1-dimensional objects.

Replacing point particles by strings from 'a Layman's guide to string theory'



Spacetime

- $\bullet\,$ We are living in a 10d Riemannian manifold ${\cal S}$
- Compactification means $\mathcal{S}=\mathbb{R}^{1,3}\times_w\mathcal{M}_6$ and \mathcal{M}_6 compact

Spacetime

- \bullet We are living in a 10d Riemannian manifold ${\cal S}$
- Compactification means $\mathcal{S}=\mathbb{R}^{1,3}\times_{w}\mathcal{M}_{6}$ and \mathcal{M}_{6} compact

Particle spectrum

• Vibrations of string $\hat{=}$ elementary particles.

Spacetime

- $\bullet\,$ We are living in a 10d Riemannian manifold ${\cal S}$
- Compactification means $\mathcal{S}=\mathbb{R}^{1,3}\times_w\mathcal{M}_6$ and \mathcal{M}_6 compact

Particle spectrum

- Vibrations of string $\hat{=}$ elementary particles.
- Every (closed) string gives rise to $|\mathbb{N}|$ elementary particles |n
 angle with masses

$$M^{2}(|n\rangle) = \frac{4}{\alpha'}(n-1)$$

where $\alpha' \ll 1$ is the *Regge slope*.

Spacetime

- $\bullet\,$ We are living in a 10d Riemannian manifold ${\cal S}$
- Compactification means $\mathcal{S}=\mathbb{R}^{1,3}\times_{w}\mathcal{M}_{6}$ and \mathcal{M}_{6} compact

Particle spectrum

- Vibrations of string $\widehat{=}$ elementary particles.
- Every (closed) string gives rise to $|\mathbb{N}|$ elementary particles |n
 angle with masses

$$M^{2}(|n\rangle) = \frac{4}{\alpha'}(n-1)$$

where $\alpha' \ll 1$ is the Regge slope.

• Unlikely to measure the massive excitations in near future

Spacetime

- \bullet We are living in a 10d Riemannian manifold ${\cal S}$
- \bullet Compactification means $\mathcal{S}=\mathbb{R}^{1,3}\times_{w}\mathcal{M}_{6}$ and \mathcal{M}_{6} compact

Particle spectrum

- Vibrations of string $\widehat{=}$ elementary particles.
- Every (closed) string gives rise to $|\mathbb{N}|$ elementary particles |n
 angle with masses

$$M^{2}(|n\rangle) = \frac{4}{\alpha'}(n-1)$$

where $\alpha' \ll 1$ is the Regge slope.

- Unlikely to measure the massive excitations in near future
- \Rightarrow Eventually focus on the massless particles

Further consequence - D-branes appear in string theory

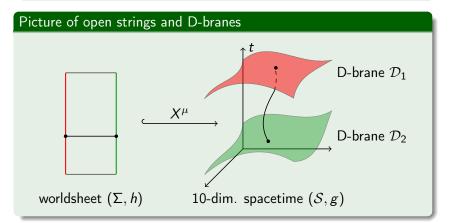
Rough picture

D-branes are places where open strings end

Further consequence - D-branes appear in string theory

Rough picture

D-branes are places where open strings end



String landscape

- $\bullet\,$ String theory has many solutions ($\sim 10^{500}$ $_{e.g.\ hep-th/0610102}$)
- String landscape := set of all solutions
- Each solution corresponds to the physics of an entire universe

String landscape

- $\bullet\,$ String theory has many solutions ($\sim 10^{500}$ $_{e.g.\ hep-th/0610102}$)
- String landscape := set of all solutions
- Each solution corresponds to the physics of an entire universe

Question

Is our universe an element of the string landscape?

String landscape

- $\bullet\,$ String theory has many solutions ($\sim 10^{500}$ $_{\rm e.g.\ hep-th/0610102}$)
- String landscape := set of all solutions
- Each solution corresponds to the physics of an entire universe

Question

Is our universe an element of the string landscape?

Answer and strategy

• A number of solutions come quite close to our universe. However, no perfect match found yet! hep-th/0502005 and many others

String landscape

- $\bullet\,$ String theory has many solutions ($\sim 10^{500}$ $_{e.g.\ hep-th/0610102}$)
- String landscape := set of all solutions
- Each solution corresponds to the physics of an entire universe

Question

Is our universe an element of the string landscape?

Answer and strategy

- A number of solutions come quite close to our universe. However, no perfect match found yet! hep-th/0502005 and many others
- General strategy:

(By use of symmetries) check every element in the landscape

String landscape

- $\bullet\,$ String theory has many solutions ($\sim 10^{500}$ $_{e.g.\ hep-th/0610102}$)
- String landscape := set of all solutions
- Each solution corresponds to the physics of an entire universe

Question

Is our universe an element of the string landscape?

Answer and strategy

- A number of solutions come quite close to our universe. However, no perfect match found yet! hep-th/0502005 and many others
- General strategy:

(By use of symmetries) check every element in the landscape

 \Rightarrow An enormous search!

The search in the string landscape



Section 2

Sheaf cohomology in string theory and the homalg package

Special solutions to string theory: the B-model hep-th/9112056

Special solutions to string theory: the B-model hep-th/9112056

The B-model . . .

is the region of the string landscape described by $_{\mbox{\tiny hep-th/0403166}}$

- $\mathcal{S} = \mathbb{R}^{1,3} \times \mathcal{M}_6$ where \mathcal{M}_6 is a CY 3-fold
- \bullet use $\mathcal{N}=(2,2)$ supersymmetric theory on \mathcal{M}_6
- string coupling $g_s
 ightarrow 0$
- focus on topological sector

• . . .

Special solutions to string theory: the B-model hep-th/9112056

The B-model . . .

is the region of the string landscape described by $_{\mbox{\tiny hep-th/0403166}}$

•
$$\mathcal{S} = \mathbb{R}^{1,3} imes \mathcal{M}_6$$
 where \mathcal{M}_6 is a CY 3-fold

- \bullet use $\mathcal{N}=(2,2)$ supersymmetric theory on \mathcal{M}_6
- string coupling $g_s
 ightarrow 0$
- focus on topological sector

• . . .

Dictionary in B-model hep-th/0208104

Physics	Mathematics	
D-brane ${\cal D}$ on ${\cal M}_6$	$\mathcal{D}\in D^{b}\left(\mathfrak{Coh}\left(\mathcal{M}_{6} ight) ight)$	
Massless string excitation s	$s\in \operatorname{Ext}^q(\mathcal{F}_ullet,\mathcal{G}_ullet)$	
between D-branes $\mathcal{F}_{ullet},\mathcal{G}_{ullet}$		

Special solutions to string theory: F-theory GUT model

Special solutions to string theory: F-theory GUT model

Step 1: Model base and GUT

General ingredients

Step 1: Model base and GUT

General ingredients

Smooth, compact toric variety X_Σ, dimX_Σ = 4

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
,
 $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

Step 1: Model base and GUT

General ingredients

Smooth, compact toric variety X_Σ, dimX_Σ = 4

 $a B_3 \subset X_{\Sigma}, \dim B_3 = 3$

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
,
 $S = \mathbb{C} [x_0, \dots, x_3, y_0, y_1]$
• $B_3 = V (\langle x_0 \rangle)$

Step 1: Model base and GUT

General ingredients

- Smooth, compact toric variety X_Σ, dimX_Σ = 4
- $a B_3 \subset X_{\Sigma}, \dim B_3 = 3$
- $S_{\text{GUT}} \subset B_3$ with $\dim S_{\text{GUT}} = 2$

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
,
 $S = \mathbb{C} [x_0, \dots, x_3, y_0, y_1]$
• $B_3 = V (\langle x_0 \rangle)$
• $S_{\text{GUT}} = V (\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \rangle)$

Step 2a: Elliptic fibration over B_3 - General Story

Step 2a: Elliptic fibration over B_3 - General Story

• For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0\left(X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}\left(i\overline{K}_{B_3}\right)\right)$.

Step 2a: Elliptic fibration over B_3 - General Story

- For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0\left(X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}\left(i\overline{K}_{B_3}\right)\right)$.
- For $[x,y,z]\in \mathbb{CP}_{2,3,1}$, $p\in B_3$ set

$$\begin{split} \mathcal{C}_p &:= \{ [x, y, z] \in \mathbb{CP}_{2,3,1} , \ Q(x, y, z, p) = 0 \} \\ Q(x, y, z, p) &:= x^2 - y^2 + xyza_1(p) + x^2 z^2 a_2(p) + yz^3 a_3(p) \\ &+ xz^4 a_4(p) + z^6 a_6(p) \end{split}$$

Step 2a: Elliptic fibration over B_3 - General Story

• For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0\left(X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}\left(i\overline{K}_{B_3}\right)\right)$.

• For
$$[x,y,z]\in \mathbb{CP}_{2,3,1}$$
, $p\in B_3$ set

$$\begin{split} \mathcal{C}_{p} &:= \{ [x, y, z] \in \mathbb{CP}_{2,3,1} , \ Q(x, y, z, p) = 0 \} \\ Q(x, y, z, p) &:= x^{2} - y^{2} + xyza_{1}(p) + x^{2}z^{2}a_{2}(p) + yz^{3}a_{3}(p) \\ &+ xz^{4}a_{4}(p) + z^{6}a_{6}(p) \end{split}$$

•
$$Y_4 := \bigcup_{p \in B_3} C_p$$
 with $\pi \colon Y_4 \to B_3$ s.t. $\pi^{-1}(p) = \mathcal{C}_p$

Step 2b: Elliptic fibration over B_3 - Our example

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

•
$$S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$$

Step 2b: Elliptic fibration over B_3 - Our example

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))].$

Step 2b: Elliptic fibration over B_3 - Our example

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))].$

• For example
$$a_1 = x_1^3 y_0 y_1, ...$$

Step 2b: Elliptic fibration over B_3 - Our example

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))].$

• For example
$$a_1 = x_1^3 y_0 y_1, ...$$

Step 3: Build A SU (5) \times U (1)_x Model Over S_{GUT}

Require that $a_2 = a_{2,1}w$, $a_3 = a_{3,2}w^2$, $a_4 = a_{4,3}w^3$, $a_6 \equiv 0$ s.t.

•
$$a_{j,k} \in H^0\left[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}\left(j \cdot \overline{K}_{B_3} - k \cdot D_{\mathsf{GUT}}\right)
ight]$$

• $a_1, a_{j,k}$ not divisable by w in S

Step 4: Matter curves

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), \ s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))].$

• For example
$$a_1 = x_1^3 y_0 y_1, ...$$

Step 4: Matter curves

• $X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), \ s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))].$
- For example $a_1 = x_1^3 y_0 y_1, ...$
- Set $a_6 \equiv 0$ and $a_2 = a_{2,1}w$, $a_3 = a_{3,2}w^2$, $a_4 = a_{4,3}w^3$ such that $a_1, a_{2,1}, a_{3,2}, a_{4,3}$ are not divisable by w.

Step 4: Matter curves

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), \ s_{B_3} \equiv x_0$$

•
$$S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$$

• For $i \in \{1, 2, 3, 4, 6\}$ pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))].$

• For example
$$a_1 = x_1^3 y_0 y_1, \dots$$

• Set $a_6 \equiv 0$ and $a_2 = a_{2,1}w$, $a_3 = a_{3,2}w^2$, $a_4 = a_{4,3}w^3$ such that $a_1, a_{2,1}, a_{3,2}, a_{4,3}$ are not divisable by w.

•
$$C_{10} := \{ p \in X_{\Sigma} , s_{B_3} = w = a_1 = 0 \}$$

• $C_{\overline{5}m} := \{ p \in X_{\Sigma} , s_{B_3} = w = a_{3,2} = 0 \}$
• $C_{5H} := \{ p \in X_{\Sigma} , s_{B_3} = w = a_{3,2} \cdot a_{2,1} - a_{4,3} \cdot a_1 = 0 \}$

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- Pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))]$ such that $a_6 \equiv 0$ and $a_2 = a_{2,1}w, a_3 = a_{3,2}w^2, a_4 = a_{4,3}w^3$.
- For example $a_1 = x_1^3 y_0 y_1, ...$
- Consider the curves $C_{10}:=\{p\in X_\Sigma\;,\;s_{B_3}=w=a_1=0\},\ldots$

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- Pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))]$ such that $a_6 \equiv 0$ and $a_2 = a_{2,1}w, a_3 = a_{3,2}w^2, a_4 = a_{4,3}w^3$.
- For example $a_1 = x_1^3 y_0 y_1, ...$
- Consider the curves $C_{10}:=\{p\in X_\Sigma\;,\;s_{B_3}=w=a_1=0\},\ldots$
- Pick $\mathcal{F} \in \text{Pic}(X_{\Sigma})$ and $\mathcal{H} \in \text{Pic}(S_{\text{GUT}}) \iota^* \text{Pic}(B_3)$.

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- Pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))]$ such that $a_6 \equiv 0$ and $a_2 = a_{2,1}w, a_3 = a_{3,2}w^2, a_4 = a_{4,3}w^3$.
- For example $a_1 = x_1^3 y_0 y_1, ...$
- Consider the curves $C_{10}:=\{p\in X_\Sigma\;,\;s_{B_3}=w=a_1=0\},\;\ldots$
- Pick $\mathcal{F} \in \text{Pic}(X_{\Sigma})$ and $\mathcal{H} \in \text{Pic}(S_{\text{GUT}}) \iota^* \text{Pic}(B_3)$.
- $H^{i}\left(C_{10}, \mathcal{H}|_{C_{10}} \otimes \mathcal{F}^{-1}|_{C_{10}} \otimes \sqrt{K_{C_{10}}}\right), \dots$ encode (massless) particle spectrum

•
$$X_{\Sigma} = \mathbb{CP}^3 \times \mathbb{CP}^1$$
, $S = \mathbb{C}[x_0, \dots, x_3, y_0, y_1]$

•
$$B_3 = V(\langle x_0 \rangle), s_{B_3} \equiv x_0$$

- $S_{\text{GUT}} = V\left(\left\langle x_0, x_0^3 + x_1^3 + x_2^3 + x_3^3 \right\rangle\right), \ w \equiv x_0^3 + x_1^3 + x_2^3 + x_3^3$
- Pick $a_i \in H^0[X_{\Sigma}, \mathcal{O}_{X_{\Sigma}}(i \cdot (3, 2))]$ such that $a_6 \equiv 0$ and $a_2 = a_{2,1}w, a_3 = a_{3,2}w^2, a_4 = a_{4,3}w^3$.
- For example $a_1 = x_1^3 y_0 y_1, ...$
- Consider the curves $C_{10}:=\{p\in X_\Sigma\;,\;s_{B_3}=w=a_1=0\},\;\ldots$
- Pick $\mathcal{F} \in \text{Pic}(X_{\Sigma})$ and $\mathcal{H} \in \text{Pic}(S_{\text{GUT}}) \iota^* \text{Pic}(B_3)$.
- $H^{i}\left(C_{10}, \mathcal{H}|_{C_{10}} \otimes \mathcal{F}^{-1}|_{C_{10}} \otimes \sqrt{K_{C_{10}}}\right), \dots$ encode (massless) particle spectrum
- \Rightarrow The actual search is then a scan over admissable fluxes (\mathcal{F}, \mathcal{H}).

Brief introduction to string theory Sheaf cohomology in string theory and the homalg package

Summary: *homalg* package = my search engine

What *homalg* can do for me:

• Toric varieties package allows to handle X_{Σ}

What *homalg* can do for me:

- Toric varieties package allows to handle X_{Σ}
- homalg package allows for the computation of Ext^q (F_•, G_•) (among many other features)

What *homalg* can do for me:

- Toric varieties package allows to handle X_{Σ}
- homalg package allows for the computation of Ext^q (F_•, G_•) (among many other features)
- homalg has Tate-resolution implemented

What *homalg* can do for me:

- Toric varieties package allows to handle X_{Σ}
- homalg package allows for the computation of Ext^q (F_•, G_•) (among many other features)
- homalg has Tate-resolution implemented
- \Rightarrow Covers many aspects of F-theory model building

What *homalg* can do for me:

- Toric varieties package allows to handle X_{Σ}
- homalg package allows for the computation of Ext^q (F_•, G_•) (among many other features)
- homalg has Tate-resolution implemented
- \Rightarrow Covers many aspects of F-theory model building

Further improvements needed:

As much performance as possible - there is a big search ahead.

Brief introduction to string theory Sheaf cohomology in string theory and the homalg package

Thank you for your attention! Questions?

